A Guidance and Navigation System for Automatic Stationkeeping in Earth Orbit

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A technique is presented for accurately stationkeeping one vehicle within a zone fixed to another. A guidance policy is developed for maintaining a limit cycle trajectory within the zone. Periodic velocity corrections are applied to minimize fuel and trajectory deviations in the presence of navigation and control uncertainties. The navigation sensors are an inertial measurement unit and a tracking radar. A square root formulation of the Kalman filter is employed to estimate the relative state and radar biases, with driving noise used to account for modeling errors. Extensive simulation results are presented to verify the design and establish key system tradeoffs. Accuracies obtained on typical trajectories were about 10 ft of estimation error and 50 ft total trajectory deviation from the nominal.

Introduction

THIS paper is concerned with the conceptual design of a guidance-and-navigation system for automatic stationkeeping of one orbiting vehicle with respect to another. For convenience in nomenclature the active vehicle, i.e., the one performing the stationkeeping maneuvers, will arbitrarily be referred to as the shuttle. The passive orbiting vehicle will be designated as the workshop. The primary emphasis is on the formulation of the necessary mathematical relations, and the testing of these relations in a detailed simulation of the over all stationkeeping problem.

Previous analyses have been principally concerned with stationkeeping of synchronous satellites[1-4] or manual techniques for stationkeeping in nonsynchronous orbits.[5,6] In the synchronous orbit studies, emphasis is on correction of lunar-solar gravity effects and solar radiation effects. In the problem studied here, the workshop orbit is nonsynchronous and the primary disturbance forces are due to Earth gravity gradients. The techniques developed for both guidance and navigation are completely automatic.

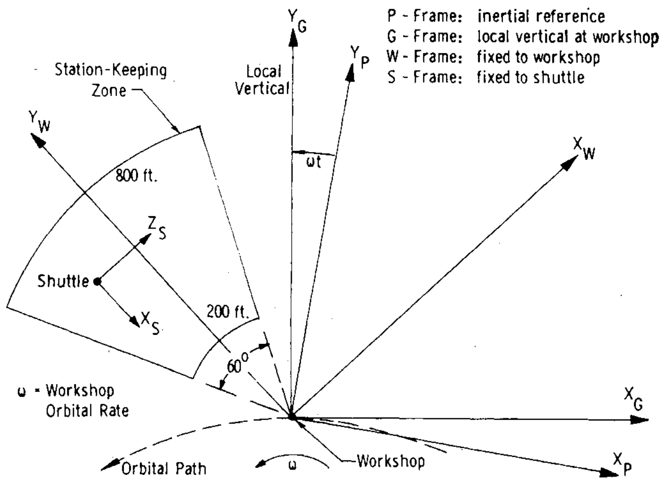

Fig. 1 Stationkeeping system relative geometry.

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Guidance System Description

The basic stationkeeping guidance objective is to maintain the shuttle vehicle within a prescribed zone fixed relative to the orbiting workshop for an extended period of time and with a minimum expenditure of fuel. The stationkeeping zone considered in this paper was based on certain assumed operational and safety requirements. The region of tracking-radar visibility was assumed to be a 60° cone with its apex at the desired docking port on the workshop. The zone was further restricted to a truncated section of the cone, extending between 200 ft and 800 ft from the workshop docking port.

In order to analyze the guidance problem, several different coordinate frames were used. Figure 1 shows the relationship of these frames and the stationkeeping zone for the case of stationkeeping above and in front of the workshop.

Dynamics of Relative Motion

Since the relative separation of the two vehicles is small, the equations of motion of the shuttle may be linearized about the workshop state. If this is done, then to first order[6,7]

$$\ddot{x}_G = 2\omega\dot{y}_G + s_{XG} \tag{1}‡$$

$$\ddot{y}_G = -2\omega\dot{x}_G + 3\omega^2 y_G + s_{YG} \tag{2}$$

$$\ddot{z}_G = -\omega^2 z_G + s_{ZG} \tag{3}$$

where s_{XG}, s_{YG}, s_{ZG} are the components of specific force, the total of the nongravitational forces applied to the vehicle per unit mass and ω is the instantaneous workshop angular velocity. The derivation of Eqs. (1–3) assume that the workshop is in a circular orbit (constant ω). It has been shown[8] that these equations are valid even if the workshop is in a slightly elliptic orbit.

The free-fall solution to the in-plane equations of motion, Eqs. (1) and (2), may take several forms. A particularly convenient form of the solution which has been presented by several authors is[6,9,10]

$$x_G(t) = x_{CG}(t) + 2b\sin(\omega t + \phi) \tag{4}$$

$$y_G(t) = y_{CG} + b\cos(\omega t + \phi) \tag{5}$$

These may be recognized as the parametric equations of an ellipse with center at (x_{CG}, y_{YG}), semimajor axis $2b$, and semiminor axis b as shown in Fig. 2. The period of the ellipse is

‡ Coordinate frames are indicated by the farthest-to-the-right subscripts, i.e., s_{XG} and x_G are in guidance coordinates (subscript G).

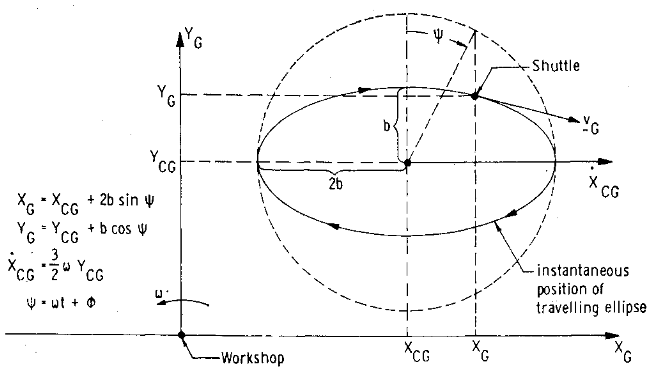

Fig. 2 Geometry of travelling ellipses in local vertical frame.

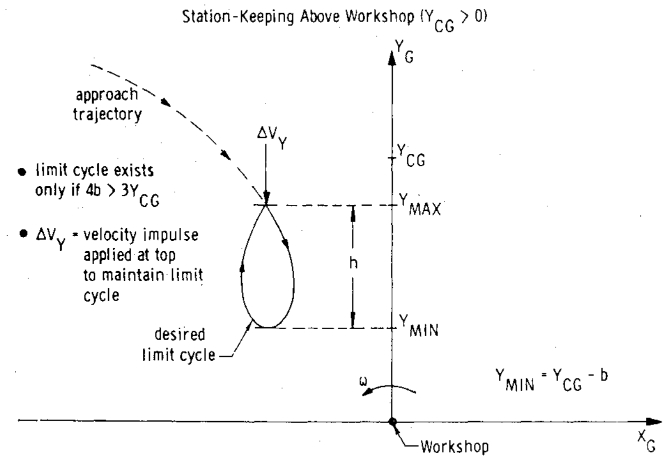

Fig. 3 Geometry for guidance limit cycles.

the period of the workshop orbit. The coordinates of the center, the semiminor axis, and phase angle ϕ are defined by the relations

$$x_{CG}(t) = x_G(0) + 2\dot{y}_G(0)/\omega + 3/2 y_{CG}\,\omega t \tag{6}$$

$$y_{CG} = 4y_G(0) - 2\dot{x}_G(0)/\omega \tag{7}$$

$$b = \{[y_{CG} - y_G(0)]^2 + [\dot{y}(0)/\omega]^2\}^{1/2} \tag{8}$$

$$\phi = \tan^{-1}\{-\dot{y}_G(0)/\omega[y_G(0) - y_{CG}]\} \tag{9}$$

The center of the ellipse travels with constant horizontal velocity $\dot{x}_{CG}$ proportional to the altitude of the center of the ellipse y_{CG}. If y_{CG} is zero, the period of the shuttle orbit is equal to the period of the workshop orbit and the ellipse center is stationary with respect to the workshop. In this case it is theoretically possible to stationkeep with no expenditure of fuel. For $y_{CG} \neq 0$, the center of the ellipse eventually drifts out of the stationkeeping zone and velocity corrections must be made to keep the shuttle within the zone.

The solution to the out-of-plane motion during free-fall is simple harmonic motion:

$$z_G(t) = z_G(0)\cos\omega t + [\dot{z}_G(t)/\omega]\sin\omega t \tag{10}$$

Since the equation of out-of-plane motion is uncoupled from the in-plane motion, it is convenient to consider the corresponding guidance problems separately.

Guidance for Controlling In-Plane Motion

The most important consideration in the ideal error-free case is that of minimizing the fuel expenditure or, equivalently, the sum of the magnitudes of the ΔV corrections. A convenient measure of performance is the ΔV/orbit. From Eq. (6) it can be seen that x_{CG} is affected instantaneously only by Y-axis velocity corrections. Likewise, from Eq. (7) it is evident that only X-axis velocity corrections change y_{CG}. Thus, in order to control a drifting ellipse, it is sufficient to use vertical velocity impulses. Horizontal velocity impulses not only waste fuel but can actually aggravate the guidance problem by increasing the relative velocity. It is shown in Ref. 11 that the policy of using only vertical impulses satisfies a sufficient condition for maintaining the shuttle within the zone as time increases without bound, and minimizes the required ΔV/orbit§ with respect to all trajectories with the same average relative altitude.

For the applications considered in this paper the average relative altitude was limited to about 800 ft and the workshop was in a 270 naut mile circular orbit. For continuous thrusting this requires $s_{YG} \simeq 0.003$ ft/sec^2, which was several orders of magnitude below the minimum thrust/weight ratio for the vehicles considered. Thus, all velocity corrections are essentially impulsive. Figure 3 depicts the geometry for a guidance scheme which uses only vertical impulsive velocity corrections applied periodically throughout the stationkeeping maneuver. The basic idea is to set up a desired cycloidal limit cycle trajectory in the guidance frame. From the figure, which shows the case of stationkeeping above the workshop, it is seen that only one velocity pulse is applied per limit cycle period. This pulse is applied down along the local vertical, and reverses the sign of the vertical velocity component. A limit cycle exists only if $\dot{x}_G(t) + \dot{x}_{CG} < 0$ at the bottom of the limit cycle.

In order to compute theoretical ΔV/orbit, the period of the limit cycle must be found. If the shuttle is at the top of the limit cycle at time t_0, the condition to maintain the limit cycle is

$$x_{CG}(t_0 + T) = x_{CG}(t_0) \tag{11}$$

where T is the limit cycle period. Equating the change in x_{CG} during the limit cycle period to the change produced by the vertical velocity impulse gives an expression for the period T in terms of the relative altitude of the ellipse center and vertical velocity just prior to correction, $\dot{y}_G'(t_0)$

$$T = 8\dot{y}_G'(t_0)/3\omega^2 y_{CG} \tag{12}$$

Note that immediately after the velocity correction, $\dot{y}_G(t_0) = -\dot{y}_G'(t_0)$. The average required fuel expenditure per workshop orbit is then:

$$\Delta V/\text{orbit} = 3\pi\,\omega y_{CG}/2 \tag{13}$$

If the altitude given in Eq. (5) is integrated over the limit cycle period, the time-average altitude is obtained as

$$\bar{y}_G = y_{CG}/4 \tag{14}$$

Thus

$$\Delta V/\text{orbit} = 6\pi\,\omega|\bar{y}_G| \tag{15}$$

which is the value required to hold the shuttle stationary above the workshop at a distance $|y_G|$.

Mechanization of In-Plane Guidance Equations

The discussion thus far has considered the ideal case of no system errors. In the actual case there will be uncertainties in the application of the velocity corrections and in knowledge of position and velocity. In order to correct for known deviations in the shuttle trajectory, a fixed-time-of-arrival guidance law is employed to compute the desired velocity correction at the top of the limit cycle. The form of the required correction is found by considering the position Eqs. (4) and (5). Assume that the shuttle is at position (x_{G1}, y_{G1}) at time t_1 and it is desired to transfer the shuttle to position

§ Fuel expenditure per workshop orbit.

(x_{G2}, y_{G2}) at time t_2. Then the required velocity corrections at time t_1 are

$$\Delta \dot{x}_{G_1} = (\omega/d)\,[(x_{G_2} - x_{G_1})\sin\theta + 2(y_{G_2} - y_{G_1})(\cos\theta - 1) + 12 y_{G_1}(1 - \cos\theta)] - \dot{x}_{G_1} \quad (16)$$

$$\Delta \dot{y}_{G_1} = (\omega/d)[2(x_{G_2} - x_{G_1})(1 - \cos\theta) + (y_{G_2} - y_{G_1})(4\sin\theta - 3\theta) - 3\theta y_{G_1}(1 - \cos\theta)] - \dot{y}_{G_1} \quad (17)$$

where

$$d = 8(1 - \cos\theta) - 3\sin\theta, \qquad \theta = \omega(t_2 - t_1) \quad (18)$$

The final time t_2 is computed from the present time and the limit cycle period T. The aim conditions (x_{G2}, y_{G2}) are stored in the computer and are precomputed based on the orientation of the desired stationkeeping zone and shuttle-vehicle characteristics. The limit-cycle period T can be computed onboard as a function of the desired minimum altitude and height of the limit cycle, rather than vertical velocity as indicated in Eq. (12). From Eq. (5) and Figs. 2 and 3 we may write

$$y_G(0) = y_{CG} - b\cos(\omega T/2) \quad (19)$$

$$\dot{y}_G(0) = -\omega b \sin(\omega T/2) \quad (20)$$

which define the altitude and altitude rate of the shuttle with respect to the workshop at the top of the limit cycle. Let h and $y_{\min}$ be the height and minimum altitude of the limit cycle as shown in Fig. 3. If the first two terms of the sine and cosine expansions are retained in Eqs. (19) and (20) and these are substituted into Eq. (12), there results

$$T = (2/\omega)[6h/(9y_{\min} + 4h)]^{1/2} \quad (21)$$

This expression has been used in the mechanization simulations in all cases except for stationkeeping almost directly in front, where the period approaches $2\pi/\omega$. Comparison with the exact value computed from Eqs. (19) and (20) has shown agreement to within 3% in all cases studied.

Guidance for Controlling Out-of-Plane Motion

It is convenient to separate the out-of-plane problem into two modes of operation. The first mode is used when the desired stationkeeping zone is centered in the workshop orbital plane and velocity corrections are employed to keep the shuttle close to the orbital plane. Since free-fall motion is centered in the orbital plane, corrections are required only to compensate for initial condition, velocity-application errors, and navigation errors. These corrections are made at the same time as the in-plane corrections and are based on the idea of driving out-of-plane position to zero at a fixed terminal time. The required velocity correction at time t_1 to obtain $z_G(t_2) = 0$ is

$$\Delta \dot{z}_{G_1} = -\omega z_{G_1}\cot\theta - \dot{z}_{G_1} \quad (22)$$

with θ defined in Eq. (18).

The second mode of operation for out-of-plane guidance is employed when the desired stationkeeping zone is not centered in the orbital plane of the workshop. In this case it is desired to maintain some prescribed minimum out-of-plane separation distance $z_{\min}$. This requirement is met by applying a velocity correction normal to the workshop plane and directed outward away from the workshop as the minimum separation distance is approached. The required velocity correction is computed as a function of the desired minimum and maximum out-of-plane separation distances $(z_{\min}, z_{\max})$ by the relation:

$$\Delta \dot{z}_G(t) = \omega[z_{\max}{}^2 - z_G{}^2(t)]^{1/2}\,\mathrm{sgn}(z_{\min}) - \dot{z}_G(t) \quad (23)$$

where sgn(·) is +1 for a positive argument and −1 for a negative argument. The value of $z_{\max}$ is found from the out-of-plane limit cycle period T_0, which is presumed to be specified a priori, according to

$$z_{\max} = 2\,|\,z_{\min}\,|\,\frac{\sin(\omega T_0/2)}{\sin(\omega T_0)}; \quad 0 < T_0 < \pi/\omega \quad (24)$$

The velocity correction is applied if the conditions $|z_G(t)| < z_{\min}$ and $\dot{z}_G(t) z_G(t) < 0$ are satisfied.

An Optimum Midcourse Correction Strategy

In the ideal case of perfect navigation data and no uncertainties in application of the required velocity correction, it has been shown that the shuttle may be maintained on the limit-cycle trajectory with only one velocity correction per limit cycle period, occurring at the top of the limit cycle. When navigation and velocity application errors are included in the system, however, the effect is to cause deviations in the shuttle trajectory from the desired limit cycle. For the vehicles considered in this paper and for the assumed error sources, the trajectory deviations were large enough to severely limit the operational capability of the system. An excessive amount of fuel was required, since it was necessary to employ large horizontal velocity corrections to stay within the zone.

For this reason, it was decided to use a small number of midcourse corrections applied to the shuttle at preselected times during the limit cycle. In order to minimize the onboard computations, it was decided to employ the same terminal guidance law used to compute velocity corrections at the top of the limit cycle, as given in Eqs. (16) and (17). In order to conserve fuel while simultaneously minimizing the deviations from the desired trajectory, it was found that only a fraction of the correction should be applied at any given time.

A procedure was developed for computing the optimum set of correction fractions for a given measurement schedule and is presented in Ref. 11. The method essentially follows that of Battin,[12] and Denham and Speyer[13] who investigated the problem of statistical optimization of midcourse-guidance corrections for space flight. The values of the correction fractions are computed prior to flight based on assumed navigation and velocity-application error statistics and are stored in the onboard computer.

Fuel Requirements vs Location and Size of Limit Cycles

One of the most important considerations in the design of the stationkeeping guidance and navigation system is that of fuel minimization. As shown previously, the fuel requirement per orbit is a strong function of average shuttle altitude and out-of-plane distance with respect to the workshop. The ΔV/orbit for in-plane limit cycles is given by the linear relation

$$\Delta V/\text{orbit} = 2\pi\omega(3y_{\min} + h) \quad (25)$$

which can be derived by substituting Eq. (12) into Eq. (19) and Eq. (20) and then using Eq. (13). For out-of-plane limit cycles the relation is

$$\Delta V/\text{orbit} = (4\pi/T_0)\left[\frac{1 - \cos\omega T_0}{\sin\omega T_0}\right] z_{\min} \quad (26)$$

which can be found by integrating Eq. (10) over a limit cycle period and then using Eq. (23).

Description of Navigation System

Basic Navigation Sensors Used

In the interest of obtaining an operational system in the shortest possible time, it was assumed that the primary navigation sensors onboard the shuttle were the rendezvous radar (RR) and inertial measurement unit (IMU) used in the Apollo lunar module. The rendezvous radar is a tracking radar which provides data on the range and range-rate of the shuttle with respect to the workshop. The radar antenna is mounted on a two-gimbal assembly. The gimbal angles provide data on the orientation of the line-of-sight from shuttle to workshop. The innermost gimbal angle is referred to for convenience as the "trunnion" angle and the outer gimbal angle is normally called the "shaft" angle. The IMU provides the basic reference coordinate frame for the guidance-and-navigation system. In addition, the IMU provides data on the changes in shuttle velocity during the periods of time that velocity corrections are applied to the vehicle. The IMU is assumed to contain three gyros and three integrating accelerometers. For preliminary design purposes it was assumed that IMU performance for the stationkeeping problem could be modeled by the errors shown in Table 1. The large alignment error given in Table 1 reflects an IMU drift over several hours from the last alignment of the stable member.

Table 1 Navigation sensor errors[a]

IMU (3-sigma values)	Rendezvous radar (3-sigma values)
SM alignment	Range:
= 20 mr/axis	200 ft (bias)
Drift-rate bias	1% range or 30 ft (random)
= 0.09° hr	Range rate:
Accelerometer bias	1 fps (bias)
= 0.02 ft/sec^2	1 fps (random)
Accel. scale factor	Gimbal angles:
= 450 ppm	16 mr/axis (bias)
Accel. quantization	6 mr/axis (random)
= 0.033 fps	Radar structure tilt:
	12 mr/axis (bias)

Extrapolation of the Vehicle State Estimates

In order to maximize accuracy in the relative state estimates, it is better to extrapolate the relative state directly via the transition matrix, rather than integrate the vehicle states separately and then take their difference. To accomplish this, it is most convenient to work in the local-vertical guidance frame (subscript G). It can then be shown that the relative state (r_G, v_G) can be extrapolated by

$$\begin{bmatrix} \mathbf{r}_G(t_n) \\ \hdashline \mathbf{v}_G(t_n) \end{bmatrix} = \Phi(t_n, t_{n-1}) \begin{bmatrix} \mathbf{r}_G(t_{n-1}) \\ \hdashline \mathbf{v}_G(t_{n-1}) \end{bmatrix} \tag{27}$$

where $\Phi(t_n, t_{n-1})$ is the transition matrix from time t_{n-1} to t_n.

The workshop state ($\mathbf{r}_{WP}$, $\mathbf{v}_{WP}$) is extrapolated forward via an onboard precision integration routine, which includes all significant disturbing forces acting on the vehicle. Earth's gravitational field harmonics and oblateness effects are taken into account in these computations. The shuttle state w.r.t. the center of the Earth ($\mathbf{r}_{SP}$, $\mathbf{v}_{SP}$) is obtained by combining the precision-integrated workshop state ($\mathbf{r}_{WP}$, $\mathbf{v}_{WP}$) with the relative state of Eq. (27).

Basic State-Vector Updating Relation

The navigation system provides up-to-date estimates of a state vector ($\mathbf{x}$) by adding to its a priori estimated value ($\mathbf{x}'$) the weighted difference between the raw measurement being incorporated ($\tilde{q}$) and its a priori estimate ($\mathbf{q}'$). A set of four individual measurements is incorporated sequentially at each updating time: range (r), range-rate ($\dot{r}$), shaft angle (θ_s), and trunnion angle (θ_t). The updating equation is

$$\mathbf{x} = \mathbf{x}' + \mathbf{w}(\tilde{q} - q') \tag{28}$$

where $\mathbf{w}$ represents the weighting vector used in the processing of the current RR measurement.

The actual quantities updated in Eq. (28), i.e., the elements of $\mathbf{x}$, are the shuttle position and velocity relative to the workshop ($\mathbf{r}_P$, $\mathbf{v}_P$) and the rendezvous radar range and range-rate bias errors (γ_r and γ_v). The arrangement of these elements in the state-vector is shown below

$$\mathbf{x}^T = [\mathbf{r}_P{}^T \mid \mathbf{v}_P{}^T \mid \gamma_r \mid \gamma_v \cdot] \tag{29}$$

Weighting-Function Computations Using a Square Root Formulation

The weighting function used in the updating process ($\mathbf{w}$) is computed continuously by an onboard minimum-variance filter. The performance criterion used in deriving the weighting function is minimization of the mean-squared errors in the quantities being estimated.

To simplify the onboard filter design, it is assumed that the shuttle state-estimation errors relative to the workshop are solely the result of shuttle state-estimation errors, i.e., the workshop state is known perfectly. This implies that only the shuttle state should be updated by the RR data. The best justification for this approximation is that it has given satisfactory results in Earth orbital rendezvous simulations,[14] and in simulated stationkeeping maneuvers (shown later in this paper) with large workshop state-estimation errors. In orbit-navigation systems operating over long periods of time, computational inaccuracies can cause the estimation-error covariance matrix used in the onboard filter to become nonpositive definite. To circumvent this problem, a square root formulation[15,16] is employed for the onboard navigation filter.

In the square root formulation a matrix W is used which is equal to the square root of the estimation-error covariance matrix E, i.e., $E = WW^T$. The desired expression for the weighting function $\mathbf{w}$ is

$$\mathbf{w} = W'W'^T\mathbf{b}/(\mathbf{b}^T W'W'^T\mathbf{b} + \overline{\alpha^2}) \tag{30}$$

where the quantity $\mathbf{b}$ represents the sensitivity of the measurement of interest to very small changes in the state vector, $\overline{\alpha^2}$ is the mean-squared measurement error, and W' is the a priori estimate of W. Likewise, it can be shown[16] that the relation for updating W after a new measurement is incorporated into the state-vector estimate is

$$W = W' - \frac{\mathbf{w}\mathbf{b}^T W'}{1 + [\overline{\alpha^2}/(\mathbf{b}^T W'W'^T\mathbf{b} + \overline{\alpha^2})]^{1/2}} \tag{31}$$

Extrapolation of the W-matrix between successive marks is accomplished by integration of the matrix differential equation

$$\dot{W} = FW + Y_D \tag{32}$$

The matrix F is used to linearly model the changes in estimation error for a vehicle falling freely in a central-force gravitational field.[15]

The significance and origin of the matrix Y_D will next be explained. In the conventional formulation of the minimum-variance estimator, the estimation-error covariance matrix (E) can be propagated during free-fall flight in a simple gravitational field by the following relation

$$\dot{E} = FE + EF^T + Q \tag{33}$$

The matrix Q represents white driving noise introduced into the navigation filter to improve the long-term navigation accuracy.[17-19] Such driving noise is often useful in circumventing problems caused by factors such as computational truncation and roundoff errors, inaccurate modeling of the navigation sensors, unaccounted-for bias errors, and approximations in the onboard filter covariance-matrix relations.

A mathematically equivalent relation to Eq. (33) in terms of W is the following[20]:

$$\dot{W} = FW + 0.5Q(W^T)^{-1} \tag{34}$$

To simplify the filter mechanization, the second term on the right-hand side of Eq. (31) is approximated by the relation

$$Y_D = 0.5 Q_D W_D^{-1} \tag{35}$$

where the subscript D is used to indicate a diagonalized matrix. The basic approximation used here is that in the computation of $Q(W^T)^{-1}$, only the diagonal terms of Q and W are considered. Note that if E and Q are diagonal, Eq. (35) is exact.

System Design Problems

As an aid in the system design, a comprehensive digital-computer simulation of the stationkeeping problem was made. Detailed models were developed for the vehicle guidance, navigation, and control systems. The particular errors modeled for the navigation sensors and the velocity-correction system are shown in Tables 1 and 2. Accelerometer quantization and bias errors are shown here because it was assumed that these instruments would be used to monitor the velocity-correction burns (i.e., issue the engine-on and engine-off commands).

In these initial studies it was assumed that the vehicles were point masses, moving about a spherical Earth with no gravitational harmonics. It was further assumed that the vehicles were sufficiently high above the Earth so that the aerodynamic forces acting on the vehicles were negligible.

Modifications to Onboard Filter W-Matrix

In early simulations using a W-matrix formulation with no driving noise for the onboard filter, it was found that after about 40 min from the start of the maneuver, the navigation errors tended to diverge as indicated in Fig. 4. This type of behavior was not unexpected, since several approximations had to be made in order to obtain a reasonable onboard filter; e.g., certain sensor bias errors were ignored, linearized error-propagation relations were used, the total relative-state estimation error was assigned to the workshop, and no computation errors were modeled.

Table 2 Velocity correction system characteristics

Propulsion system	Application errors
Shuttle weight = 200,000 lb	Autopilot dead band = 1° (random orient.)
Lateral thrust = 1600 lb (jet pair)	Thruster alignment = 3°/axis (3-sigma)
Longitude thrust = 3200 lb (jet pair)	Burn monitoring interval = 0.1 sec
F/m Lat = 0.25 ft/sec²	Accel. quantization = 0.033 fps
F/m Long = 0.50 ft/sec²	Accel. bias = 0.006 ft/sec² (3-sigma)

[a] No ΔV penalties for rotational control during translational maneuvers (c.g. offsets).

In an attempt to improve navigation-system accuracy, the idea of reinitializing the W-matrix, which has been used by Muller et al.[14,21] in Apollo orbital-rendezvous mission phases, was next tried. The particular scheme used here was to reset the W-matrix to its initial value at specified time intervals. This technique, by periodically raising the onboard filter weighting functions, did prevent the divergence of navigation errors.

An alternate scheme investigated was to limit the W-matrix diagonals to preselected minimum values. The idea here was to place lower limits on the radar weighting functions so that new measurements could influence the state estimates. This technique also prevented the long-term buildup of navigation error. The over-all navigation-system performance in the cases studied was comparable to that for the reinitialization scheme.

The third and most successful technique found for improving the accuracy of the onboard filter was to include a driving-noise term in the relation used to extrapolate the W-matrix between successive updatings (60 sec apart), as described

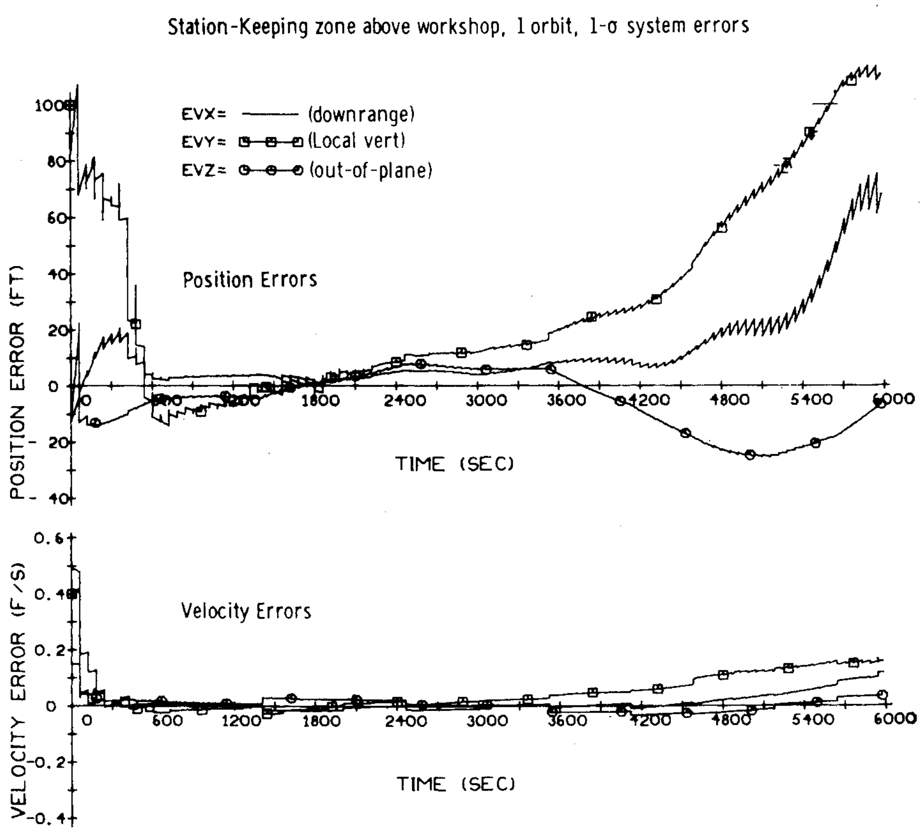

Fig. 4 Navigation errors with no onboard filter modifications.

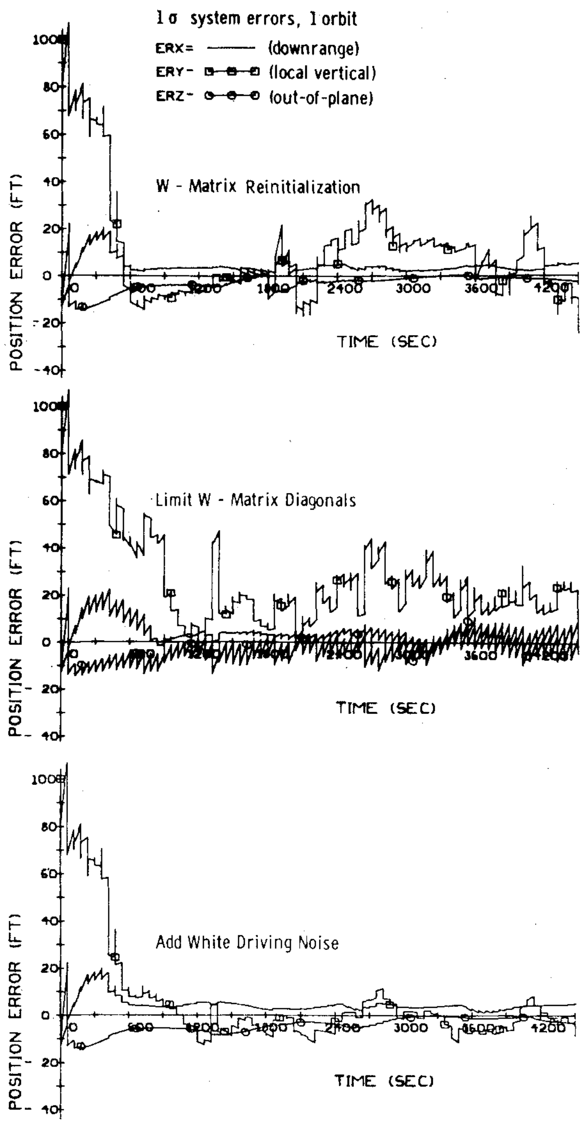

Fig. 5 Effect of onboard filter modifications on position errors.

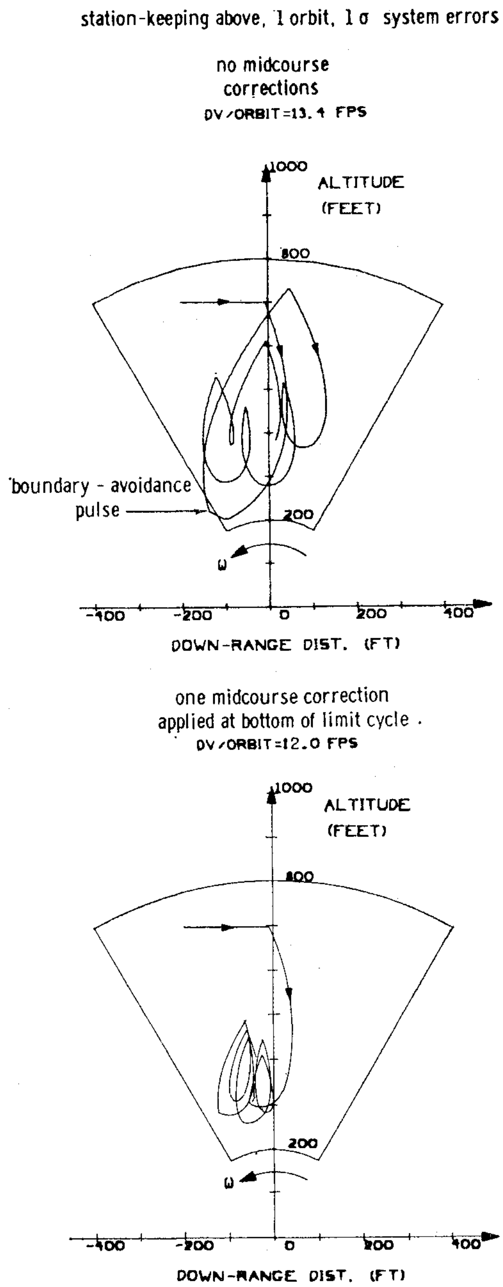

Fig. 6 Effect of guidance midcourse corrections on stationkeeping trajectory.

earlier in the paper. The navigation accuracies obtained by this technique are shown in Fig. 5 for the same case considered in Fig. 4. For comparison purposes the corresponding errors are shown for cases where W-matrix reinitialization and lower limits on W-matrix diagonals were used. It is evident that the driving-noise modification to the onboard filter clearly gives the best performance.

Effect of Midcourse Velocity Corrections on System Performance

As a result of the assumed errors in the navigation sensors (Table 1) and in application of the guidance velocity corrections (Table 2), it is difficult to maintain the shuttle within the stationkeeping zone with only one velocity correction per limit cycle period. This is demonstrated in Fig. 6, which shows trajectory profiles for a typical case of stationkeeping above the workshop for one orbit. As shown in the figure, the use of one midcourse correction at the bottom of the limit cycle reduced the fuel expenditure by about 10% and significantly reduced the deviations from the desired limit cycle.

A study was made to assess the effect of various midcourse correction schemes on the fuel requirements and the position and velocity deviations at the top of the limit cycle. The results are summarized in Table 3 for an inplane limit cycle above the workshop. Both statistical and actual error data are presented, using one and three equally-spaced corrections. A full correction indicates that the velocity increments computed from Eqs. (14) and (15) are used without modification. The optimum fractions presented in the table were found by the methods of Ref. 11. It should be mentioned that the values of the fractions are strongly influenced by the designer's choice of the relative weighting given the terminal error penalty vs the penalty for velocity corrections.

Both the statistical and actual data indicate that the use of midcourse corrections significantly reduces the terminal position and velocity deviations. The best over-all performance occurred for the case of one optimum midcourse correction, which reduced the rms terminal position error from 169 ft to about 60 ft at a cost of only 0.1 fps/orbit. Terminal position deviations of the order of 200 ft can result in pulses to avoid the boundary (Fig. 6), which are very costly from a fuel standpoint.

Stationkeeping System Performance Data

Data on the expected performance of the stationkeeping system under typical conditions are presented in this section

Table 3 Comparison of actual and statistical performance using midcourse corrections (1200 sec limit cycle period, equally-spaced corrections)

Number of Corrections	Correction times	Correction fractions	Monte Carlo runs[a] rms ΔV/orbit, fps	Monte Carlo runs[a] rms terminal errors pos., ft	Monte Carlo runs[a] rms terminal errors vel.,m fps	Statistical run[b] rms ΔV/orbit fps	Statistical run[b] rms terminal errors pos., ft	Statistical run[b] rms terminal errors vel., fps
0	...	...	1.9	205	0.36	2.2	168	0.41
1	600	1	2.8	60	0.08	3.1	65	0.2
1	600	0.84[c]	2.0	64	0.05	2.3	61	0.17
3	300, 600, 900	1, 1, 1	3.6	28	0.20	3.8	34	0.18
3	300, 600, 900	0.65,[c] 0.70[c], 0.78[c]	2.5	40	0.11	2.3	31	0.16

[a] Monte Carlo data based on ten limit cycles.
[b] Details on generation of statistical data in Ref. 6.
[c] Optimum values.

based on simulation studies. The assumed duration of the stationkeeping phase was two orbits of the workshop around the Earth (approximately 12,000 sec in the simulated cases). Sensor errors were modeled as in Table 1, velocity-correction errors as in Table 2, and state-estimation errors at the start of the stationkeeping phase as in Table 4. All actual errors were set at their 1σ value. In all cases shown here, both range and range-rate bias errors are estimated and, except for Fig. 5, driving noise is included in the extrapolation of the onboard filter W-matrix.

Stationkeeping Zone above Workshop and Centered in Its Orbital Plane

The trajectory of the shuttle w.r.t. the workshop in a typical situation for this case is shown in Fig. 7. As can be seen, the guidance-and-navigation system keeps the shuttle well within the stationkeeping zone at a cost of 9.4 fps ΔV per orbit. The navigation-system position and range bias estimation errors for the same simulation run are shown in Fig. 8. The position estimation errors are less than 20 ft. Accuracies of this magnitude are possible because of the ability of the navigation-system to estimate the radar bias errors.

Stationkeeping Zone in Front or Off to Side

Simulations were run for cases where the stationkeeping zone was centered off to the side or in front of the workshop. The shuttle was maintained in a zone directly out to the side at a cost of about 4.5 fps/orbit and in a zone directly in front at a cost of about 0.4 fps/orbit. The navigation and guidance errors were essentially the same in both cases as those obtained with the zone directly above the workshop. The major exception was the horizontal (line-of-sight) estimation error which reached as much as 100 ft with the zone placed directly in front. In this situation, the movement normal to the line-of-sight was so slow that the radar biases could not be accurately estimated. Note, however, that this does not cause an increase in fuel expenditure.

Table 4 Initial condition estimation errors

Workshop			
1-sigma est. errors	X (down range)	Y (vertical)	Z (cross track)
Position	50,000 ft	4000 ft	550 ft
Velocity	3 fps	55 fps	1 fps
Shuttle w.r.t. workshop			
1-sigma est. errors	X (down range)	Y (vertical)	Z (cross track)
Relative position	100 ft	100 ft	20 ft
Relative velocity	0.4 fps	0.4 fps	0.15 fps

Summary of Results and Conclusions

A guidance-and-navigation system has been presented for automatically stationkeeping a shuttle vehicle in a zone arbitrarily located with respect to an orbiting workshop. The guidance system maintains the shuttle on a steady-state limit-cycle trajectory within the stationkeeping zone by application of periodic velocity corrections to the vehicle. The guidance policy which has been developed provides essentially a minimum-fuel trajectory under ideal conditions. Small midcourse corrections are used to compensate for errors in the navigation information and in the execution of the stationkeeping maneuvers.

The accurate relative position and velocity information required in the stationkeeping problem was obtained by pro-

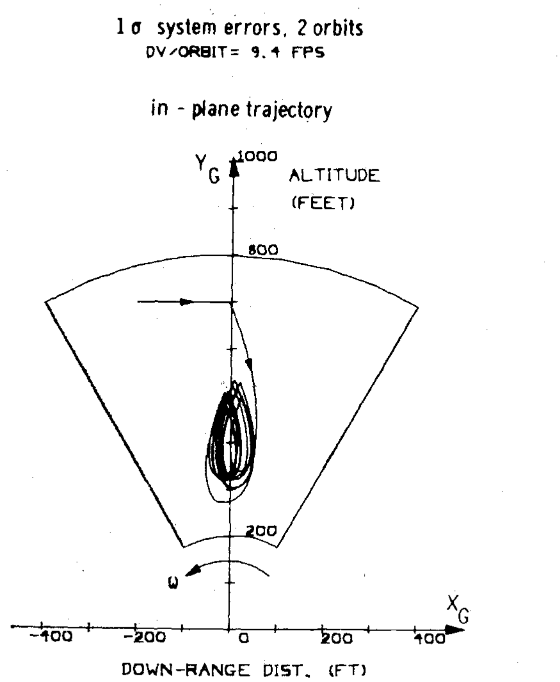

Fig. 7 Shuttle trajectory w.r.t. workshop—stationkeeping zone above workshop.

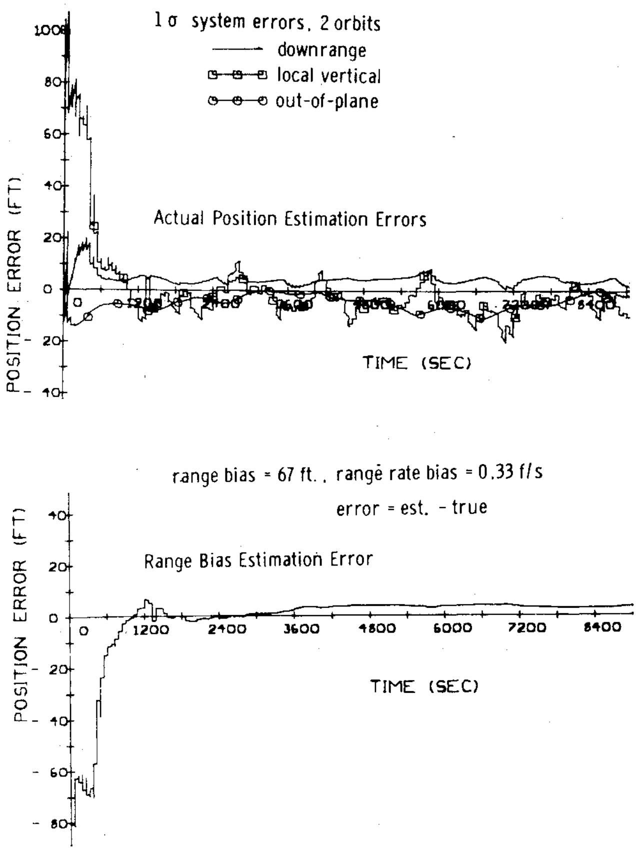

Fig. 8 Navigation system performance for stationkeeping above workshop.

cessing the tracking-radar and inertial measurement unit data in a minimum-variance onboard filter, using a square root formulation to avoid computation errors. In order to obtain satisfactory performance for the over-all system, it was necessary to estimate both the range and range-rate bias errors of the tracking radar. Significant improvements in the long-term performance accuracy of the onboard filter were obtained by introducing a driving noise into the *W*-matrix propagation relations to correct for various necessary filter-design approximations.

Simulation results demonstrate that the guidance-and-navigation system performed in a satisfactory manner for cases where the desired stationkeeping zone was located above, in front, and off to the side of the workshop. The required fuel expenditure, as measured by the ΔV/orbit, varied from a maximum of about 10 fps/orbit with the zone above the workshop, to a minimum of less than 1 fps/orbit with the stationkeeping zone in front of the workshop.

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